Test of Mathematics for University Admission, 2021 Paper 2
Worked Solutions

Version 1.0, March 2022

Contents

Introduction for students

These solutions are designed to support you as you prepare to take the Test of Mathematics for University Admission. They are intended to help you understand how to answer the questions, and therefore you are strongly encouraged to **attempt the questions first** before looking at these worked solutions. For this reason, each solution starts on a new page, so that you can avoid looking ahead.

The solutions contain much more detail and explanation than you would need to write in the test itself – after all, the test is multiple choice, so no written solutions are needed, and you may be very fluent at some of the steps spelled out here. Nevertheless, doing too much in your head might lead to making unnecessary mistakes, so a healthy balance is a good target!

There may be alternative ways to correctly answer these questions; these are not meant to be 'definitive' solutions.

The questions themselves are available on the 'Preparing for the test' section on the Admissions Testing website.

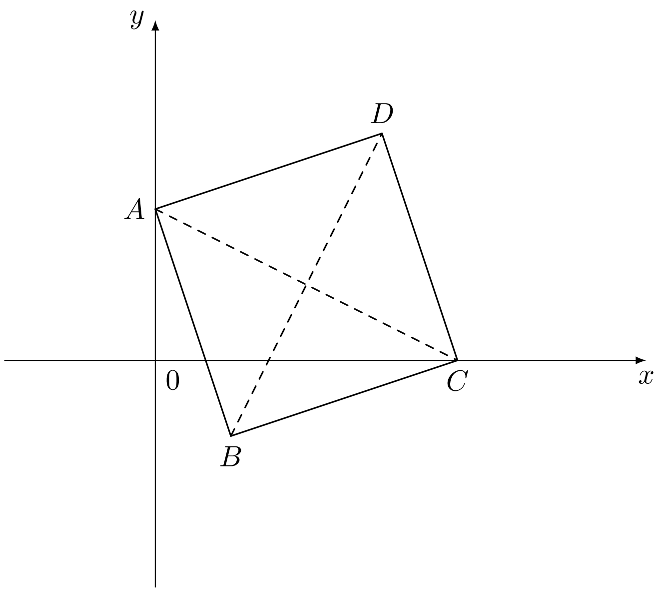
Question 1

We write the integral as powers of x and then integrate:

$$\begin{aligned}
\int_1^4 \left(3\sqrt{x} + \frac{4}{x^2}\right) \mathrm{d}x &= \int_1^4 3x^{\frac{1}{2}} + 4x^{-2} \,\mathrm{d}x \\
&= \left[\frac{3x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + \frac{4x^{-1}}{-1}\right]_1^4 \\
&= \left[2x^{\frac{3}{2}} - 4x^{-1}\right]_1^4 \\
&= (2 \times 8 - 4 \times \tfrac{1}{4}) - (2 \times 1 - 4 \times 1) \\
&= 15 - (-2) \\
&= 17
\end{aligned}$$

Question 2

We can sketch the situation:



We know that the two diagonals of the square bisect each other at right angles, so this question requires us to find the perpendicular bisector of AC.

The midpoint of AC is $\left(\frac{0+4}{2}, \frac{2+0}{2}\right) = (2, 1)$, which is the centre of the square.

The gradient of AC is $\frac{0-2}{4-0} = -\frac{1}{2}$, so the gradient of BD is 2.

The equation of BD is therefore $y - 1 = 2(x - 2)$, which rearranges to give $y = 2x - 3$, which is option E.

Question 3

The probability is given as a fraction in its lowest terms.

The probability that a student does not wear glasses is $\frac{11}{15}$, so the ratio of glasses to non-glasses is $4 : 11$, in its lowest terms.

Therefore the number of glasses/non-glasses wearers could be 4 and 11 (total 15), or 8 and 22 (total 30), or ...

Necessary means that it follows as a consequence.

Therefore I and III are not necessary (there could be 22 non-glasses wearers or the class could contain 15 students.

The total must be a multiple of 15, and so the number of students is divisible by 3.

The correct answer is therefore C.

Question 4

I $bc = 200$, a is a factor of 200, a is a factor of b, so not a counterexample.

II $bc = 16$, a is a factor of 16, a is not a factor of b and a is not a factor of c, so this is a counterexample.

III $bc = 84$, a is a factor of 84, a is not a factor of b but a is a factor of c, so not a counterexample.

The correct option is therefore C.

Question 5

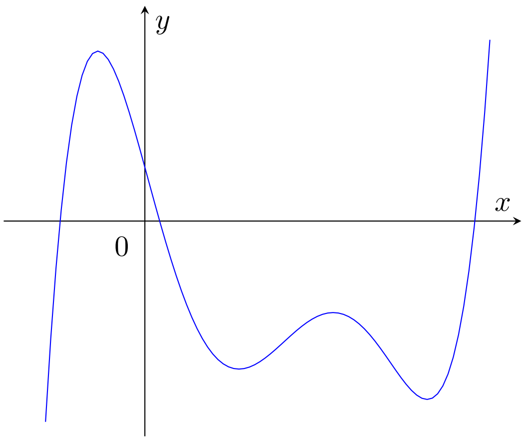
Line A is a known identity, so this is fine.

Line B rearranges the identity to get $\cos^2 x = 1 - \sin^2 x$ and then takes the square root of both sides. However, when we take the square root, we always get the positive (or zero) root, but $\cos x$ is negative for some values of x. So this is where the error is introduced.

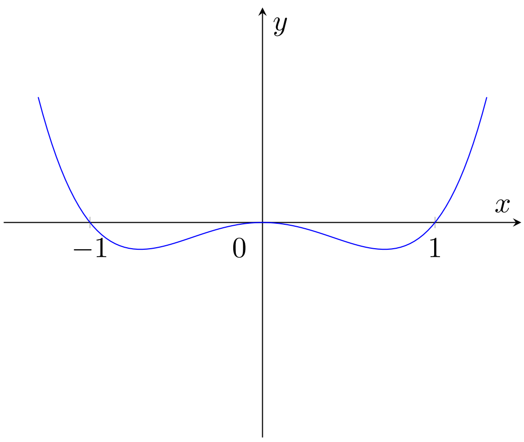
Another way of seeing this is to start with the nonsensical $0 = 4$ of line E, and substitute $x = \pi$ in each line to find the first one which fails; it is, indeed, line B, as this gives $-1 = 1$.

Question 6

If $f(x) = 0$ has exactly three roots then there must be a stationary point between the first and second root, and another stationary point between the second and third roots, but there may be other stationary points. For example, here is a sketch of a degree 5 polynomial $y = f(x)$ which has $f(x) = 0$ for three real values of x but $f'(x) = 0$ for five real values of x:

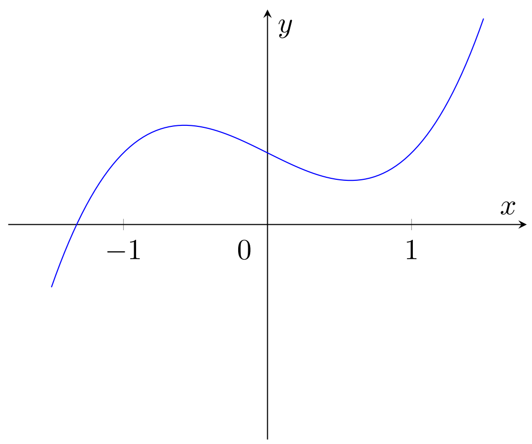


A second example is $f(x) = x^4 - x^2$. Here, the graph of $y = f(x)$ touches the x-axis at $x = 0$, so $f(x) = 0$ has exactly three real roots $x = -1$, 0, and 1, but three stationary points $x = \pm\frac{1}{\sqrt{2}}$ and $x = 0$:



So P is not sufficient for Q.

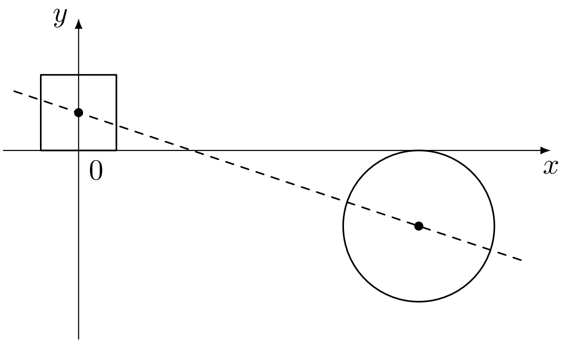
It is also not necessary, for example $f(x) = x^3 - x + 1$ has exactly two stationary points but only one real root:



So option D is correct.

Question 7

We start by drawing the square and the circle. Since the straight line bisects the area of each shape, it must pass through their centres, as shown.



This straight line therefore passes through the centre of the square at $(0, 1)$ and the centre of the circle at $(9, -2)$. The gradient of this line is then $\frac{-2-1}{9-0} = -\frac{1}{3}$, so the line has equation $y = -\frac{1}{3}x + 1$.

The line meets the x-axis where $y = 0$, so $0 = -\frac{1}{3}x + 1$, giving $x = 3$.

Hence the correct answer is B.

Question 8

If $\text{p}(a) = \text{p}(b) = c$, say, then if the graph of $y = \text{p}(x)$ (strictly) increases as it leaves (a, c) towards $x = b$, but then it must decrease again to get back to (b, c), so there must be a stationary point between $x = a$ and $x = b$. Similarly, if the graph (strictly) decreases as it leaves (a, c), it must then increase again, which also results in a stationary point. The only other possibility is that the graph of $y = \text{p}(x)$ is constant, in which case $\text{p}'(x) = 0$ throughout. In all cases, $(*)$ holds, so $\text{p}(a) = \text{p}(b)$ is sufficient for $(*)$.

However, it is not necessary for $(*)$: if $\text{p}(x) = x^2$ and $a = -1$, $b = 2$, then this satisfies $(*)$ (with $c = 0$) but $\text{p}(a) \neq \text{p}(b)$.

Thus the correct option is C.

Question 9

I Every cubic crosses the x-axis, so this condition is sufficient for $\mathrm{f}(x) = 0$ to have a real root. This also follows from III below if we take u to be a very large positive number and v to be a very large negative number.

II Consider, for example, $\mathrm{f}(x) = x^2 + 1$. This quadratic has $\mathrm{f}'(x) = 2x$ so $\mathrm{f}'(0) = 0$, but there is no real x for which $x^2 + 1 = 0$. So this condition is not sufficient.

III As $\mathrm{f}(u)\,\mathrm{f}(v) < 0$, we must have $\mathrm{f}(u) < 0$ and $\mathrm{f}(v) > 0$ or the other way round; either way, the graph of $y = \mathrm{f}(x)$ has values above and below the x-axis. Since $\mathrm{f}(x)$ is a polynomial, it must therefore cross the x-axis, and so the equation $\mathrm{f}(x) = 0$ has a real root. (If we did not know that $\mathrm{f}(x)$ was a polynomial, this would not be sufficient; consider $\mathrm{f}(x) = \frac{1}{x}$ for example.)

The correct option is therefore C.

Question 10

To be a counterexample, we need the antecedent (the 'if' part) to be true but the consequent (the 'then' part) to be false.

n	n is prime	a_n multiple of 3 or 5
1	false	–
2	true	true
3	true	true
4	false	–
5	true	false
6	false	–
7	true	false

Therefore $n = 5$ and $n = 7$ are both counterexamples, and the smallest counterexample is $n = 5$, which is option E.

Question 11

We start by checking that each step works.

Line (II) follows from line (I) by expanding the brackets.

The algebraic expression in line (III) expands as $x^2+2xy+y^2-4xy$, so it follows from line (II).

Line (IV) follows from line (III) by multiplying by x^2y^2. Since $x^2y^2 \geq 0$, the inequality keeps the same direction, so this is a valid deduction.

However, how does line (V) follow from line (IV)? It assumes that $a = xy(x+y)$ and $b = xy$ in order to make the deduction.

This argument has therefore shown that *if* $a = xy(x+y)$ and $b = xy$, *then* $a^2 - 4b^3 \geq 0$, or even more explicitly: *if* there exist real numbers x and y such that $a = xy(x+y)$ and $b = xy$, *then* $a^2 - 4b^3 \geq 0$. However, the problem posed is the converse: 'show that *if* $a^2 - 4b^3 \geq 0$, *then* there exist real numbers x and y such that $a = xy(x+y)$ and $b = xy$.

So this attempt has proved the converse, though the argument would have been slightly better presented if the student had explicitly stated 'assume that x and y are real numbers with $a = xy(x+y)$ and $b = xy$'.

The correct option is therefore C.

Question 12

I We can think of the integral $\int_0^x \mathrm{f}(t)\,\mathrm{d}t$ as being the (signed) area under the graph of $y = \mathrm{f}(x)$ from 0 to x. Since $\mathrm{f}(x) \geq \mathrm{g}(x)$ for all $x \geq 0$, the area under the graph of $y = \mathrm{f}(x)$ will also be greater than that under the graph of $y = \mathrm{g}(x)$, so this statement is true.

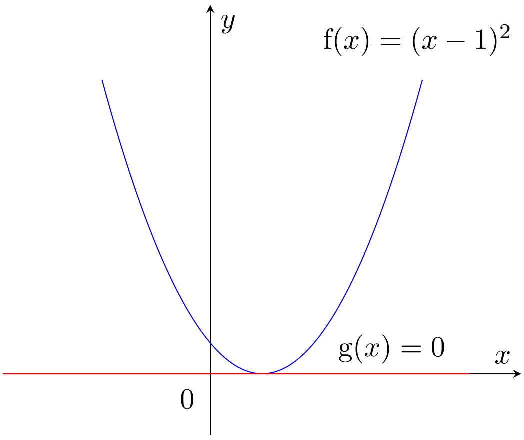
You might be concerned about values of x for which $\mathrm{g}(x)$ is negative. We could therefore demonstrate this algebraically as well.

We have

$$\int_0^x \mathrm{f}(t)\,\mathrm{d}t - \int_0^x \mathrm{g}(t)\,\mathrm{d}t = \int_0^x (\mathrm{f}(t) - \mathrm{g}(t))\,\mathrm{d}t.$$

Since $\mathrm{f}(t) - \mathrm{g}(t) \geq 0$ for all $x \geq 0$, we are integrating a non-negative function, and so the integral must be non-negative, that is to say, we must have $\int_0^x \mathrm{f}(t)\,\mathrm{d}t \geq \int_0^x \mathrm{g}(t)\,\mathrm{d}t$.

II It is not clear why $\mathrm{f}(x) \geq \mathrm{g}(x)$ should imply that the gradient (derivative) of $\mathrm{f}(x)$ is greater than that of $\mathrm{g}(x)$, and it is relatively straightforward to find a counterexample. Here is one:



In this example, $\mathrm{f}(x) \geq \mathrm{g}(x)$ for all x, but $\mathrm{f}'(0) < 0$ while $\mathrm{g}'(x) = 0$ for all x.

III This is not necessarily true, as we could add a constant to $\mathrm{g}(x)$ without changing $\mathrm{g}'(x)$. For example, we could have $\mathrm{f}'(x) = \mathrm{g}'(x) = 1$ for all x, but $\mathrm{f}(x) = x$ while $\mathrm{g}(x) = x + 1$, so $\mathrm{f}(x) < \mathrm{g}(x)$ for all x.

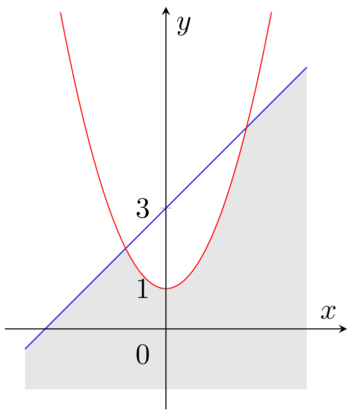
The correct option is therefore B.

Question 13

Manipulating inequalities algebraically requires care. For example, we cannot (in general) multiply two inequalities together and obtain a valid inequality (which is what statement II has done). Neither can we subtract two inequalities (which is where statement I comes from).

In this case, one approach is to sketch the region R described by the pair of inequalities and then determine whether the inequalities in statements I–III are satisfied in the whole of R. Another approach is to search for counterexamples for each statement, that is, values of x and y that do not satisfy the statement but do satisfy the original inequalities. This, however, might be harder to do, and if the inequalities are true, it may take a while to discover this. So let us sketch the given inequalities.

The inequality $y - x < 3$ can be rearranged as $y < x + 3$, so it is the region below $y = x + 3$. Similarly, $y - x^2 < 1$ can be rearranged as $y < x^2 + 1$, so it is the region below the quadratic $y = x^2 + 1$. The region R is therefore the shaded area shown here (excluding the boundaries):



We can now consider the three statements.

I It is clear that this is not true in general, as the region R extends in all directions. For example, the point $(10, 0)$ lies in R (it is easy to check that this satisfies the original inequalities: $0 - 10 < 3$ and $0 - 10^2 < 1$) but $x = 10$ does not satisfy $-1 < x < 2$.

II If we take $x = 0$, the inequality becomes $y^2 < 3$, but $(0, -2)$ is in R and $(-2)^2 = 4$. So this statement is not true for all points in R.

III We could take, for example, $(6, 6)$ as a point in R with $y \geq 5$, so this statement is not true in general.

The correct option is therefore A.

Question 14

Writing $X = 2^x$ and $Y = \log_2 y$ gives the simultaneous equations

$$pX + Y = 2$$
$$X + Y = 1$$

Subtracting gives $(p-1)X = 1$, so as long as $p \neq 1$, we have $X = \frac{1}{p-1}$ and so $Y = \frac{p-2}{p-1}$.

Now the equation $\log_2 y = \frac{p-2}{p-1}$ gives a real value for y for any value of $p \neq 1$ (specifically $y = 2^{(p-2)/(p-1)}$), but the equation $2^x = \frac{1}{p-1}$ has a solution if and only if $\frac{1}{p-1} > 0$, which is if and only if $p > 1$.

Therefore the original simultaneous equations have a real solution (x, y) if and only if $p > 1$, which is option C.

Question 15

Approach 1

We start by completing the square to find the centre and radius of the circle:

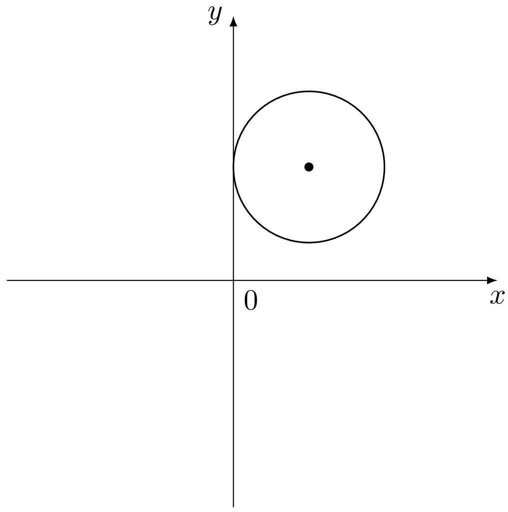
$$(x+\tfrac{1}{2}a)^2-(\tfrac{1}{2}a)^2+(y+\tfrac{1}{2}b)^2-(\tfrac{1}{2}b)^2+c=0$$

which gives

$$(x+\tfrac{1}{2}a)^2+(y+\tfrac{1}{2}b)^2=(\tfrac{1}{2}a)^2+(\tfrac{1}{2}b)^2-c$$

so the circle has centre $(-\frac{1}{2}a,-\frac{1}{2}b)$ and radius $\sqrt{(\frac{1}{2}a)^2+(\frac{1}{2}b)^2-c}$.

Now let us sketch a circle tangent to the y-axis:



We can see that the circle is tangent to the y-axis if and only if its radius is equal to the distance of its centre from the y-axis. This is *not* the same as saying the radius is equal to the x-coordinate of the centre, as the centre may have a negative x-coordinate. As an equation, this condition is then

$$\sqrt{(\tfrac{1}{2}a)^2+(\tfrac{1}{2}b)^2-c}=\left|-\tfrac{1}{2}a\right|$$

which, on squaring, is equivalent to

$$(\tfrac{1}{2}a)^2+(\tfrac{1}{2}b)^2-c=(\tfrac{1}{2}a)^2$$

(This is equivalent to the previous equation because both sides of the first equation are non-negative.)

Subtracting $(\frac{1}{2}a)^2$ from both sides gives $\frac{1}{4}b^2-c=0$ or $b^2=4c$, so this condition is necessary and sufficient for the circle to be tangent to the y-axis, which is option B.

Approach 2

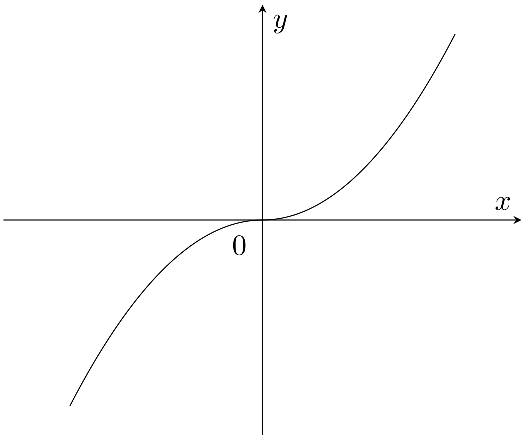
The circle is tangent to the y-axis, $x=0$, if and only if the pair of simultaneous equations $x^2+ax+y^2+by+c=0$ and $x=0$ has a repeated root.

Substituting $x=0$ into the circle equation gives $y^2+by+c=0$, which has discriminant b^2-4c. So there is a repeated root if and only if $b^2-4c=0$, which is equivalent to option B.

Question 16

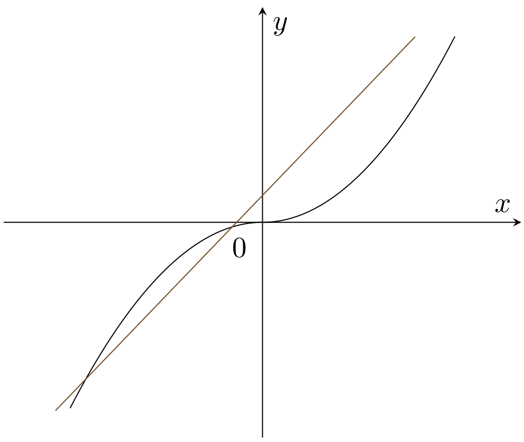
Approach 1: graph sketching

We are looking for the intersections of the graphs of $y = x\,|x|$ and $y = px + q$. The latter is just a straight line graph, while the former is $y = x^2$ when $x \geq 0$ and $y = -x^2$ when $x < 0$:



This graph looks quite similar to the graph of $y = x^3$: it has a point of inflexion at the origin. Any straight line will meet this graph at least once: if $q > 0$, then it will meet it for some $x > 0$, and if $q < 0$, then it will meet it for some $x < 0$. A straight line can meet the graph twice if it is tangent to the curve somewhere, and it can meet it three times, for example $y = x$.

But a straight line cannot meet the curve four times. It can meet the curve at most twice for $x \geq 0$, and at most twice for $x < 0$ (as each half of the curve is a quadratic, and a straight line and quadratic can meet at most twice). But if a straight line meets the quadratic twice in the region $x < 0$, then it must have $q > 0$, for example:



It can then meet the curve only once in the region $x > 0$. The same applies if it meets the curve twice in the region $x > 0$. And if the line passes through the origin, then it can meet the curve in at most two other locations.

So the correct answer is option E.

Approach 2: algebraic

If x is a solution, then either $x \leq 0$ and $-x^2 = px + q$ or $x > 0$ and $x^2 = px + q$; we can rewrite these as $x^2 + px + q = 0$ and $x^2 - px - q = 0$ respectively.

If $q < 0$, then the quadratic equations both have positive discriminant. The first quadratic has roots of opposite signs (as $\sqrt{p^2 - 4q} > p$), so there is exactly one negative solution. The second quadratic (corresponding to $x > 0$) may have 0, 1 or 2 positive solutions, giving 1, 2 or 3 solutions in total.

If $q > 0$, exactly the opposite is true, and we still end up with 1, 2 or 3 solutions.

If $q = 0$, then $x = 0$ is a root of both quadratics, and there are 0, 1 or 2 other roots.

Thus the possible values are 1, 2 and 3.

Examples with these numbers are: $p = q = 0$; $p = -1$, $q = 0$, and $p = -2$, $q = 1$ respectively.

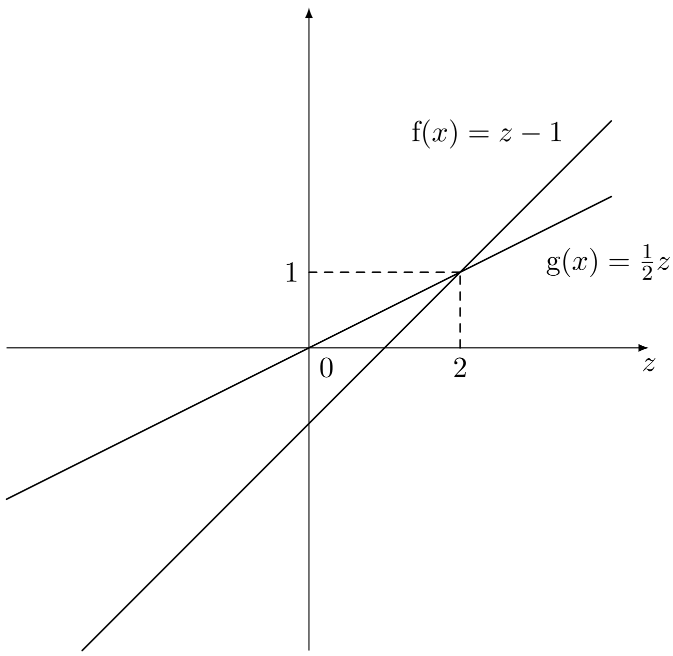
Question 17

The expressions for $\mathrm{f}(x)$ and $\mathrm{g}(x)$ look quite complicated, so we start by simplifying them using the laws of logarithms:

$$\begin{aligned}
\mathrm{f}(x) &= \log_2\bigl(\log_2 \sqrt{x}\,\bigr) \\
&= \log_2(\tfrac{1}{2}\log_2 x) \\
&= -1 + \log_2(\log_2 x) \\
\mathrm{g}(x) &= \log_2\bigl(\sqrt{\log_2 x}\,\bigr) \\
&= \tfrac{1}{2}\log_2(\log_2 x)
\end{aligned}$$

We now see that both $\mathrm{f}(x)$ and $\mathrm{g}(x)$ can be written as a function of $\log_2(\log_2 x)$. As $x > 1$, $\log_2 x > 0$ and $\log_2(\log_2 x)$ can take any real value.

Let us write $z = \log_2(\log_2 x)$, so that we have $\mathrm{f}(x) = z - 1$ and $\mathrm{g}(x) = \frac{1}{2}z$. We can sketch a graph showing these two functions of z:



We can calculate the intersection of $y = z - 1$ and $y = \frac{1}{2}z$, obtaining $z = 2$, $y = 1$.

We can then see from the graph that $\mathrm{f}(x) \geq \mathrm{g}(x)$ when both are at least 1, and $\mathrm{f}(x) \leq \mathrm{g}(x)$ when both are at most 1, so either $\mathrm{f}(x) \geq \mathrm{g}(x) \geq 1$ or $\mathrm{f}(x) \leq \mathrm{g}(x) \leq 1$ (or both), which is option F.

(Note that the '**or**' in the options is inclusive 'or', so we are allowed to have both of the inequalities being true.)

Question 18

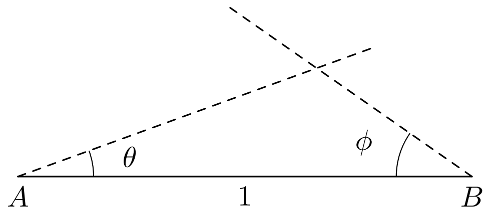
Since we are only given a condition on $\sin A$, the angle A could be acute or obtuse, and similarly for angle B.

Let us write $\theta = \sin^{-1} x$ and $\phi = \sin^{-1} y$ for the acute angles; the respective obtuse angles are then $180^\circ - \theta$ and $180^\circ - \phi$.

As we are given $x < y$, we have $\theta < \phi$.

We can now consider the four possible cases.

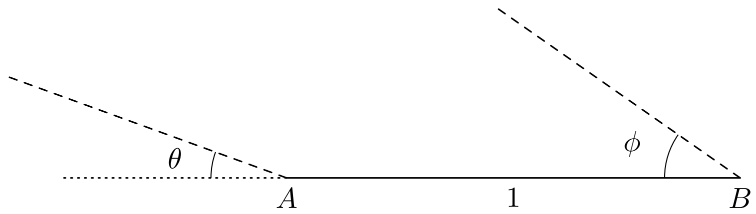
- Case 1: both A and B are acute. Then we have this situation:



  We therefore know the three angles of the triangle ABC (as given two, we automatically know the third) and one side, and this determines the triangle uniquely (regarding congruent triangles as the same).

  So we always obtain exactly one triangle from this case.

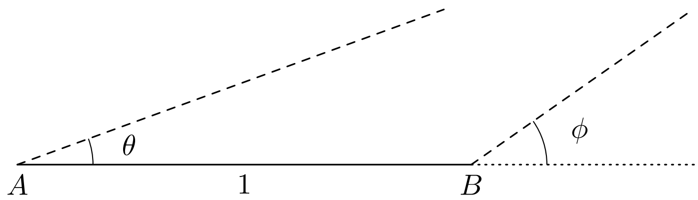
- Case 2: A is obtuse and B is acute. Then $A = 180^\circ - \theta$ and we have this situation:



  Since $\theta < \phi$, the two lines (rays) from A and B shown in this diagram will never meet. Alternatively, we could note that $A + B = 180^\circ - \theta + \phi$, and since $\phi - \theta > 0$, this means that the angles A and B sum to more than 180°.

  Therefore this case can never produce a triangle.

- Case 3: A is acute and B is obtuse. Then $B = 180^\circ - \phi$ and we have this situation:



  Since $\theta < \phi$, the two lines (rays) from A and B shown will eventually meet, and so we get a unique triangle. Alternatively, we have $A + B = \theta + 180^\circ - \phi = 180^\circ - (\phi - \theta)$, and since $\phi - \theta > 0$, this means that the angles A and B sum to less than 180°, so they can be two angles of a triangle.

  So we always obtain exactly one triangle from this case.

- Case 4: A and B are both obtuse. This is impossible in a triangle.

  Therefore this case can never produce a triangle.

In summary, cases 1 and 3 always produce a unique triangle, and cases 2 and 4 never produce a triangle, so the student can always draw exactly two different triangles, regardless of the values of x and y.

Therefore the correct answer is C.

Question 19

We can rewrite the given equation as

$$\sin\theta\sqrt{(1+\sin\theta)(1-\sin\theta)}+\cos\theta\sqrt{(1+\cos\theta)(1-\cos\theta)}=0$$

and then multiply out the brackets to give

$$\sin\theta\sqrt{1-\sin^2\theta}+\cos\theta\sqrt{1-\cos^2\theta}=0$$

which simplifies to

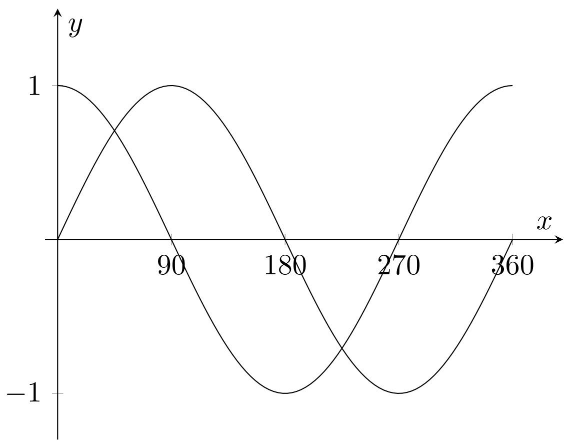
$$\sin\theta\sqrt{\cos^2\theta}+\cos\theta\sqrt{\sin^2\theta}=0.$$

It would be easy at this point to make the mistake of simplifying $\sqrt{\cos^2\theta}$ to $\cos\theta$, but this would only work in the case that $\cos\theta \ge 0$ as square roots are always positive. We can, though, write $|\cos\theta|$ for this square root: we always have $\sqrt{x^2}=|x|$. Therefore we can rewrite the equation as

$$\sin\theta\,|\cos\theta|+\cos\theta\,|\sin\theta|=0.$$

If both $\sin\theta$ and $\cos\theta$ are positive, then this becomes $2\sin\theta\cos\theta=0$, which cannot be true. Likewise, if both $\sin\theta$ and $\cos\theta$ are negative, this becomes $-2\sin\theta\cos\theta=0$, which again cannot be true. However, if one is positive and one is negative, say $\sin\theta>0$ and $\cos\theta<0$, this becomes $-\sin\theta\cos\theta+\sin\theta\cos\theta=0$, which is certainly true. The same holds if $\sin\theta<0$ and $\cos\theta>0$. Finally, if either $\sin\theta=0$ or $\cos\theta=0$, the equation is also satisfied.

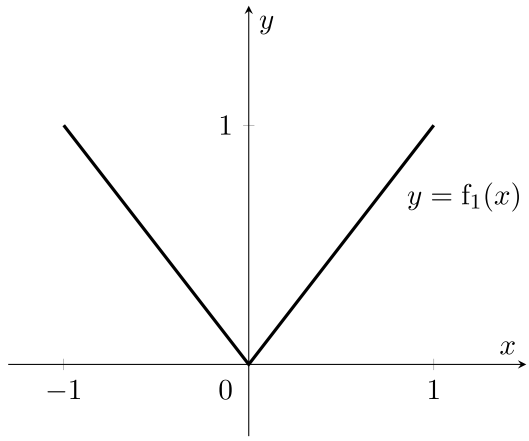
We therefore need to count the number of given angles for which $\sin\theta$ and $\cos\theta$ have opposite signs or one of them is zero. A quick sketch helps here:



We see that the relevant ranges are $90^\circ \le \theta \le 180^\circ$ and $270^\circ \le \theta \le 360^\circ$; $x=0^\circ$ would also work, but is not one of the angles in the given list. There are 91 angles in each of the two ranges (remembering to include both endpoints), so there are 182 angles in total, which is option F.

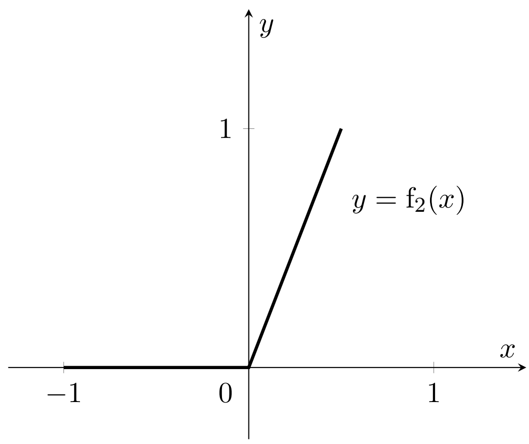
Question 20

It is helpful to get a feeling for what this sequence of functions looks like, so we will sketch the first few. We are only interested in the range from -1 to 1 (because that is what the integral asks for). The first function, $\mathrm{f}_1(x) = |x|$ is standard:

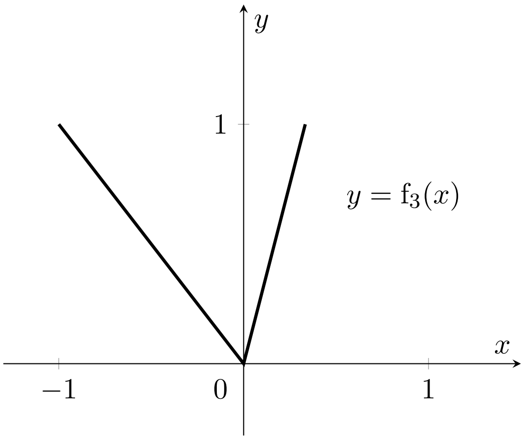


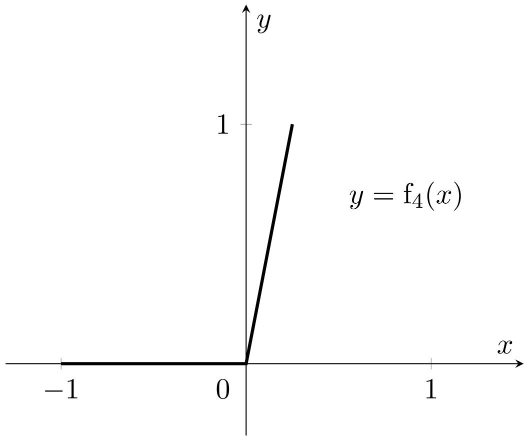
Note that when $x < 0$, this is the graph of $y = -x$, and when $x \geq 0$, this is the graph of $y = x$.

We next have $\mathrm{f}_2(x) = |\mathrm{f}_1(x) + x|$. We can calculate this on the two regions $x < 0$ and $x \geq 0$ separately: when $x < 0$, we get $\mathrm{f}_2(x) = |-x+x| = 0$, and when $x > 0$, we get $\mathrm{f}_2(x) = |x+x| = 2x$, giving this graph:



We can determine $\mathrm{f}_3(x)$ and $\mathrm{f}_4(x)$ in a similar way:





Here, the graphs for $x \geq 0$ are $y = 3x$ and $y = 4x$ respectively.

We can see a pattern forming: when n is odd, the graph of $y = f_n(x)$ looks like $y = f_3(x)$, with $y = -x$ for $x < 0$ and $y = nx$ for $x \geq 0$, and when n is even, it looks like $y = f_4(x)$, with $y = 0$ for $x < 0$ and $y = nx$ for $x \geq 0$.

We are interested in $f_{99}(x)$, which looks like $y = f_3(x)$: it is $y = -x$ for $x < 0$ and $y = 99x$ for $x \geq 0$. We can calculate the integral $\int_{-1}^{1} f_{99}(x)\,dx$ by finding the area under the graph, which is just the sum of two triangles.

The area between $x = -1$ and $x = 0$ is $\frac{1}{2} \times 1 \times 1$ and the area between $x = 0$ and $x = 1$ is $\frac{1}{2} \times 1 \times 99$, so the integral equals $\frac{1}{2} \times 1 + \frac{1}{2} \times 99 = \frac{1}{2} \times 100 = 50$; the correct option is therefore E.

Cambridge Assessment
Admissions Testing
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA
United Kingdom

Admissions tests support: **admissionstesting.org/help**